# INFLUENCE OF HEAT CAPACITY OF A VESSEL WALL AND OF THE HEAT CURRENTS IN IT ON THE CHARACTERISTICS OF CONVECTION IN A CLOSED VOLUME

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UDC 536.24

The author analyzes the influence of the wall heat capacity and of the heat currents in it on the characteristics of convective heat transfer in a cylindrical vessel with hemispherical ends.

The investigation of convective heat transfer in closed volumes is the subject of a considerable number of studies [1-4], but in most of these in analyzing the phenomena the authors do not account for the finite heat capacity of the vessel wall, although cases arise where this is considerable, especially if one is dealing with unsteady heating of a liquid. In the literature, in particular in a number of heat-transfer handbooks [5, 6], some engineering methods are given for calculating the influence of the vessel wall. As a rule, the analysis is based on solving the one-dimensional heat-conduction equation, in which one accounts for the resistance to heat flux of the contents of the vessels, the heat capacity, and the external convective resistance. In [7] in a one-dimensional formulation the authors examined the unsteady problem for a planar wall, and here the thermal resistance of the vessel wall is neglected, which is quite permissible for metal containers, but the heat capacity is accounted for. The corresponding results are formulated for an infinite cylinder and a sphere. There are practically no results on the influence of heat capacity of the vessel wall and of heat fluxes in it on the temperature stratification, the heat transfer, and other local characteristics of convection in a closed volume. The results given in the present paper can be used to calculate these factors in the study of convective heat transfer in a closed volume.

We investigate convective heat transfer in a cylindrical vessel of height H with hemispherical ends of radius R, completely filled with a liquid (the case H = 2R corresponds to a sphere). The vessel is formed by a thin-walled shell of constant thickness $\delta$. We consider that the direction of action of the mass forces coincides with the vertical axis of the vessel, that the physical properties of the liquid are independent of temperature, and that the flow and temperature fields are axisymmetric. To the outer surface of the vessel at time t = 0 we apply a constant uniformly distributed heat flux. The vessel shell is considered thin ($\delta/R \ll 1$), and we therefore neglect variation of temperature with radius in the shell. The system of dimensionless equations in the variables vorticity $\omega$, stream function $\psi$, and temperature $\theta$ to determine the variation with time of the shell temperature, and of the flow and temperature fields in the liquid has the form:

$$\frac{\delta}{R}\,\frac{\rho_w c_w}{\rho c}\,\frac{\partial\theta}{\partial t}=\frac{\delta}{R}\,\frac{k_w}{k}\,\frac{\partial^2\theta}{\partial z^2}+(1+\delta/R)-q_n(z,\ t), \tag{1}$$

$$\frac{\delta}{R}\,\frac{\rho_w c_w}{\rho c}\,\frac{\partial\theta}{\partial t}=\frac{\delta}{R}\,\frac{k_w}{k}\,\frac{1}{\sin\varphi}\,\frac{\partial}{\partial\varphi}\left(\sin\varphi\,\frac{\partial\theta}{\partial\varphi}\right)+(1+\delta/R)^2-q_n(\varphi,\ t), \tag{2}$$

$$\frac{\partial\omega}{\partial t}+\frac{1}{r}\,\frac{\partial\psi}{\partial r}\,\frac{\partial\omega}{\partial z}-\frac{1}{r}\,\frac{\partial\psi}{\partial z}\,\frac{\partial\omega}{\partial r}=\mathrm{Ra}^*\mathrm{Pr}\,\frac{1}{r}\,\frac{\partial\theta}{\partial r}+\mathrm{Pr}\left(\frac{\partial^2\omega}{\partial r^2}+\frac{3}{r}\,\frac{\partial\omega}{\partial r}+\frac{\partial^2\omega}{\partial z^2}\right), \tag{3}$$

$$\omega=\frac{1}{r^2}\,\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r^3}\,\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\,\frac{\partial^2\psi}{\partial z^2}, \tag{4}$$

$$\frac{\partial\theta}{\partial t}+\frac{1}{r}\,\frac{\partial\psi}{\partial r}\,\frac{\partial\theta}{\partial z}-\frac{1}{r}\,\frac{\partial\psi}{\partial z}\,\frac{\partial\theta}{\partial r}=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\,\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2}. \tag{5}$$

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Moscow Institute of Economics and Statistics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 5, pp. 760-764, May, 1985. Original article submitted April 3, 1984.

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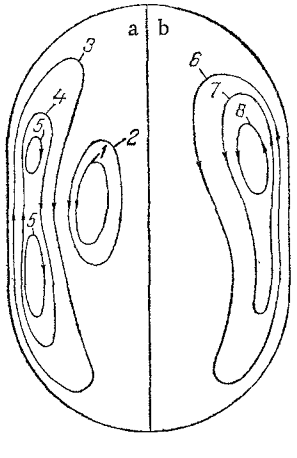


Fig. 1

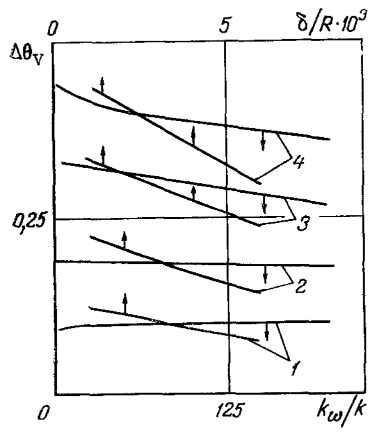


Fig. 2

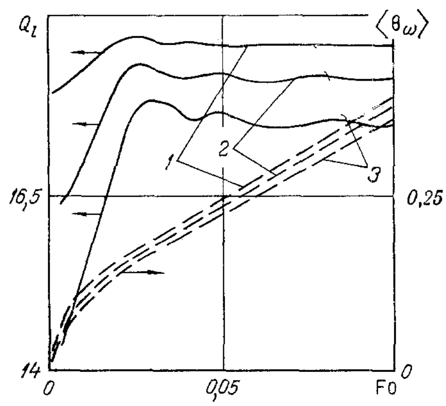


Fig. 3

Fig. 1. Stream lines (a — $\delta/R = 10^{-3}$, b — $6\cdot10^{-3}$): 1) $\psi = 1$; 2) 2; 3) −3; 4) −9; 5) −12; 6) −4.1; 7) −12.4; 8) −16.5.

Fig. 2. Dependence of $\Delta\theta_V$ on $\delta/R$ and $k_w/k$: 1) Fo = 0.02; 2) 0.03; 3) 0.05; 4) 0.1.

Fig. 3. Dependence of the mean wall temperature and the heat flux from the wall to the liquid on the Fourier number: 1) $\delta/R = 10^{-3}$; 2) $3\cdot10^{-3}$; 3) $6\cdot10^{-3}$.

At time zero the liquid is at rest, and the liquid temperature is constant and is $T_0$. The system (1)-(5) was solved with the aid of an implicit difference scheme of variable direction type on a nonuniform mesh with monotonic approximation of the convective terms [8].

We seek a parametric relation for the dimensionless temperature in the form

$$\theta = f(r,\ z,\ \mathrm{Fo},\ H/R,\ \delta/R,\ k_w/k,\ \rho_w c_w/(\rho c),\ \mathrm{Pr},\ \mathrm{Ra}^*).$$

Allowance for the finite heat capacity of the shell leads to the appearance of three additional similarity parameters $k_w/k$, $\rho_w c_w/(\rho c)$, $\delta/R$, compared with the case where the heat capacity is not accounted for. In the present study we analyze the influence of the shell heat capacity and its heat currents at Rayleigh number $\mathrm{Ra}^* = 10^6$, Prandtl number Pr = 1 and H/R = 3. We note that for a vessel of this configuration the Rayleigh number is close to $10^6$ for which one has a maximum of temperature stratification in quasisteady conditions. The values of the similarity variables $k_w/k$, $\rho_w c_w/(\rho c)$, $\delta/R$ were varied in the ranges $1 \leqslant k_w/k \leqslant 200$, $0.7 \leqslant \rho_w c_w/(\rho c) \leqslant 6$, $10^{-3} \leqslant \delta/R \leqslant 6\cdot10^{-5}$.

Initially we carried out a series of calculations to evaluate the degree of influence of the parameters $\delta/R$, $k_w/k$, $\rho_w c_w/(\rho c)$ on the temperature stratification, the heat transfer, and the level of liquid motion. This investigation showed that the relative vessel shell thickness $\delta/R$ had the greatest influence. On the whole the nature of the flow with increasing relative shell thickness can be described by some decrease of the level of motion, by attenuation of the secondary flows, and by a decrease of the thermal stratification. Figure 1 shows pictures of stream lines for two values of $\delta/R$ at the value Fo = 0.03 [$k_w/k = 125$, $\rho_w c_w/(\rho c) = 4.96$]. In the central part of the vessel with the thinnest shell a secondary flow develops which presses the main flow to the vessel walls. For an increase of wall thickness by a factor of 6 the stream line picture (Fig. 1b) at a given Fourier number corresponds only to the time of the secondary flow beginning. The intensities of the main and the secondary flows can be judged by comparing the maximum values of the vertical velocity component at the section H/2 on the vessel axis for the main and the secondary flows. For $\delta/R = 10^{-3}$ and $6\cdot10^{-3}$ these ratios were $168/141 \approx 1.2$ and $50/36 \approx 1.4$, respectively. As regards the quasistationary regime Fo ⩾ 0.1, the values of Nusselt number $\mathrm{Nu} = 1/\langle\theta_w - \theta_m\rangle$ practically coincide, while the maximum values of the stream function amplitude, which describe the convection intensity, differ by less than 2%.

As a result of a series of calculations for $\mathrm{Ra}^* = 10^6$, Pr = 1, H/R = 3, $\rho_w c_w/(\rho c) = 4.96$ we obtained the dependence of the vertical temperature differences $\Delta\theta_V = \theta(0, H) - \theta(0, 0)$, describing the thermal stratification of the liquid, on the relative vessel shell thickness $\delta/R$ (in this case $k_w/k = 125$) and on the ratio of the thermal conductivities of the wall and the liquid ($\delta/R = 3\cdot10^{-3}$) for various values of Fourier number (Fig. 2). It turned out that the vertical temperature differences $\Delta\theta_V$ for a given Rayleigh number are practically independent of the value of the parameter $k_w/k$ up to the time Fo ≃ 0.03, which corresponds approximately to the maximum secondary flow intensity. We note that the maximum intensity of the main circulatory flow occurs in this case for Fo = 0.02. The influence of the parameter $k_w/k$ begins to show up at the time Fo = 0.035, i.e., immediately after the attenuation of the first secondary flow, and for Fo = 0.1 the difference in $\Delta\theta_v$ reaches 23%. Typically, with efflux of time the role of the parameter $k_w/k$ becomes more noticeable, since the time to establish quasistationary conditions of liquid heating, according to the vertical differences $\Delta\theta_v$, is close in this case to the time to establish this regime with heat transfer by conduction (Fo ∿ 1) [6]. This is due to attenuation of convection in heating from above, as a result of which the heat transfer by conduction becomes comparable with convective heat transfer [9].

In contrast with the parameter $k_w/k$, the quantity $\delta/R$ influences the value of $\Delta\theta_v$ practically immediately after the heat supply commences, and then it has a somewhat greater influence than $k_w/k$ on the value of $\Delta\theta_v$. A decrease of $\delta/R$ by a factor of 6 leads to an increase of the vertical temperature differences with Fo = 0.1 by 45%, which is approximately 13° for the value $q_wR/k_w$ = 100°K.

Figure 3 shows a decrease with time of the heat flux coming from the wall to the liquid $Q_l=\iint\limits_S \mathrm{grad}\,\theta\cdot\mathbf{n}dS$ for various values of $\delta/R$. The derivatives $\partial\theta/\partial r$ and $\partial\theta/\partial z$ were calculated from three points on the nonequilibrium mesh. The broken lines in Fig. 3 show the corresponding dependence of the mean wall temperature $\langle\theta_w\rangle$ on the Fourier number. The dependence of $Q_l$ on the Fourier number for various values of $\delta/R$ is very complex. At first, up to a time corresponding to the appearance of secondary flow at a given $\delta/R$, the quantity $Q_l$ increases sharply, and then, after some fluctuations, goes smoothly to the quasistationary regime. The fluctuations of the quantity $Q_l$ are associated with the periodic appearance of secondary flows that attenuate with time. We note also that for low values of Fo, i.e., in the heat-conduction regime, a considerable part of the heat flux coming from outside to the wall goes to increase the heat content of the wall, for vessels with $\delta/R \geqslant 3\cdot10^{-3}$.

## NOTATION

H, vessel height; R, cylinder radius; $\delta$, shell thickness; r, z, cylindrical coordinates; $\varphi$, angle in the spherical coordinate system; $\rho_w$, $\rho$, densities of the shell and the liquid; $c_w$, c, specific heats of the shell and the liquid; $k_w$, k, thermal conductivities of the shell and the liquid; $T_0$, initial liquid temperature; $q_w$, specific heat flux to the vessel wall; $a$, thermal diffusivity; $\mathbf{n}$, unit vector normal to the vessel wall and directed into the liquid; $\omega$, dimensionless vorticity function; $\psi$, dimensionless stream function; $\theta = (T - T_0)k/(q_wR)$, dimensionless temperature of the liquid and the wall; $a/R$, velocity scale; $Ra^* = g\beta q_wR^4/(\nu a k)$, Rayleigh number; $Pr = \nu/a$, Prandtl number; $Fo = at/R^2$, Fourier number; $Nu = 1/\langle\theta_w - \theta_m\rangle$, Nusselt number; $Q_l$, total heat flux from the wall to the liquid at a given time; $\Delta\theta_v = \theta(0, H) - \theta(0, 0)$, vertical temperature difference on the vessel axis; $\langle\theta_w\rangle$, mean wall temperature; $q_n$, specific heat flux from the wall to the liquid.

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# METHODS OF SOLVING CONVECTION AND HEAT-TRANSFER PROBLEMS IN REGIONS WITH BOUNDARIES THAT VARY IN FORM OVER TIME

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UDC 532.516

A method of numerical solution of nonsteady two-dimensional Navier–Stokes equations in regions with curvilinear moving boundaries is proposed. As an example, the solution of the problem of melting with convection in the liquid phase is presented.

The need to investigate convective problems in regions of complex geometry has given rise to a stream of new numerical methods of calculation. At the same time, the question of the accuracy of these methods, the minimal calculation time, and the demands which they make remains to be resolved. For nonsteady problems with varying geometry of the region, the calculation time is one of the basic factors in selecting the numerical integration scheme.

Two approaches to the solution of this kind of problem exist: the first is associated with the interpolation of the boundary positions with respect to the points of the calculation grid and the second with matching the grid lines with the boundaries. As shown in [1], because of the large rounding errors associated with the presence of grid points very close to the boundary, it is preferable to use the second approach, the more so in that in this case the specification of the boundary conditions is considerably simplified.

A general description of the method of coordinate transformation for conservative and nonconservative systems of partial differential equations of first and second order was given in [2]. This method was developed in [3] for the problem of heat conduction with a single mobile boundary. Extensive results on the use of the method of automatic numerical construction of a curvilinear coordinate system of general form with grid lines coinciding with all the boundaries of a body of arbitrary form were published in [4]. Because of its generality, this approach is especially applicable. However, in the case when the boundaries of the region change form, additional iterations are necessary at each computational step to reconstruct the coordinate system, which may lead to significant increase in the time (i.e., cost) of the calculations.

In the present work, a simple numerical method of solving nonsteady heat- and mass-transfer problems in regions with moving curvilinear boundaries is proposed, on the basis of transforming the physical region to rectangular form. This transformation is not associated with the solution of a system of Poisson equations for the coordinates, i.e., does not require additional consumption of computer resources.

Consider a physical region consisting in the general case of our moving curvilinear boundaries (Fig. 1). The corresponding transformed region will have fixed rectilinear boundaries. The coordinate-transformation law is determined in the form

$$\xi = \frac{x - x_1}{x_2 - x_1}, \quad \eta = \frac{y - y_1}{y_2 - y_1}, \quad \tau = (\mathrm{const})\, t, \tag{1}$$

where

$$x_1 = x_1(y,\ t), \quad x_2 = x_2(y,\ t), \\ y_1 = y_1(x,\ t), \quad y_2 = y_2(x,\ t), \tag{2}$$

and therefore

$$\xi = \xi(x,\ y,\ t), \quad \eta = \eta(x,\ y,\ t). \tag{3}$$

Moscow Power Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 5, pp. 765-771, May, 1985. Original article submitted March 12, 1984.

  